

# Triaxial Galaxy Clusters

Importance for Weak  
Gravitational Lensing Mass  
Measurements

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Cluster Weighing Workshop, MPE  
July 31, 2008

# Dark Matter Halos

- ◆ Collisionless Cold Dark Matter: Navarro, Frenk, & White 1997
- ◆ Universal density profile (NFW)

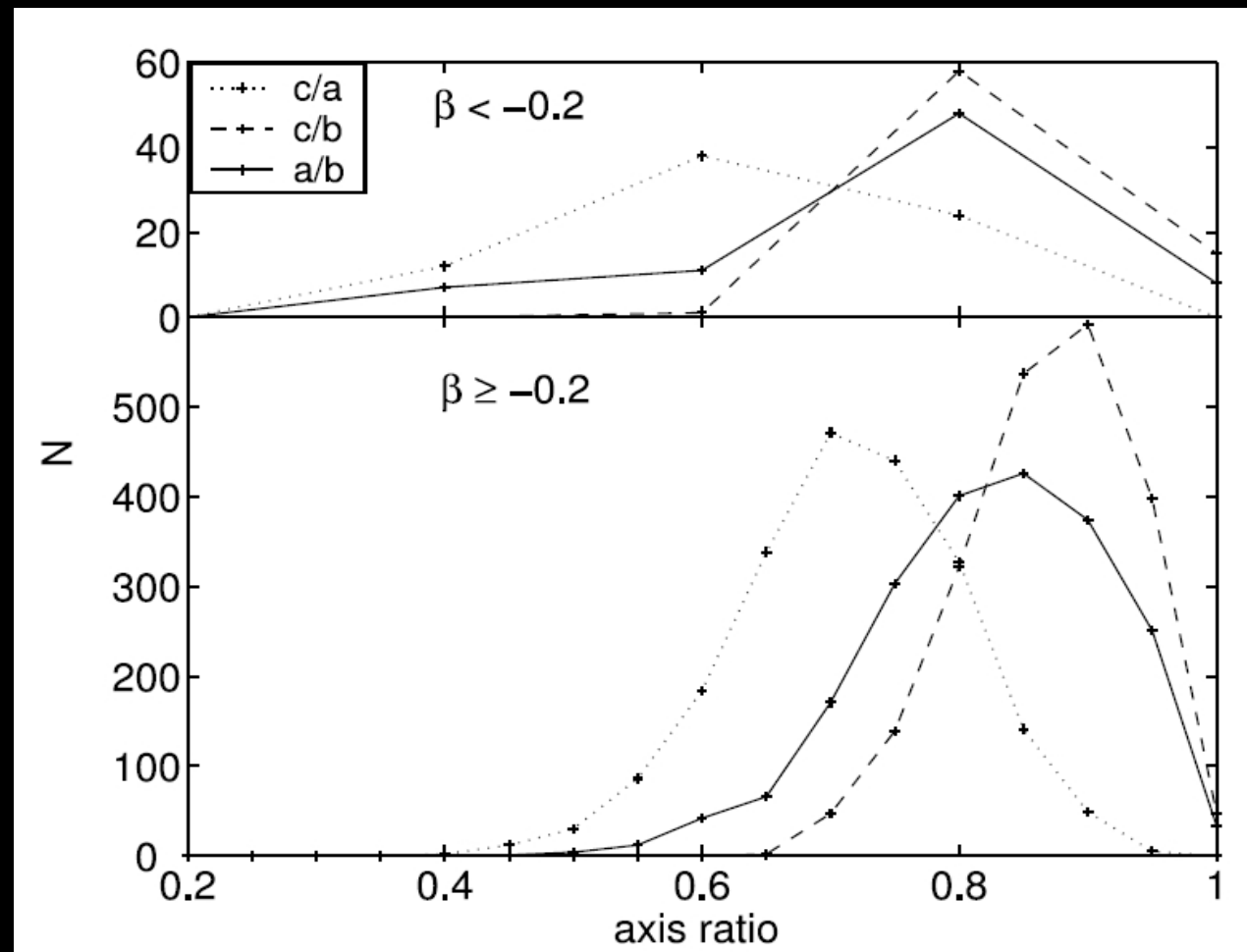
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- ◆ Significant triaxiality

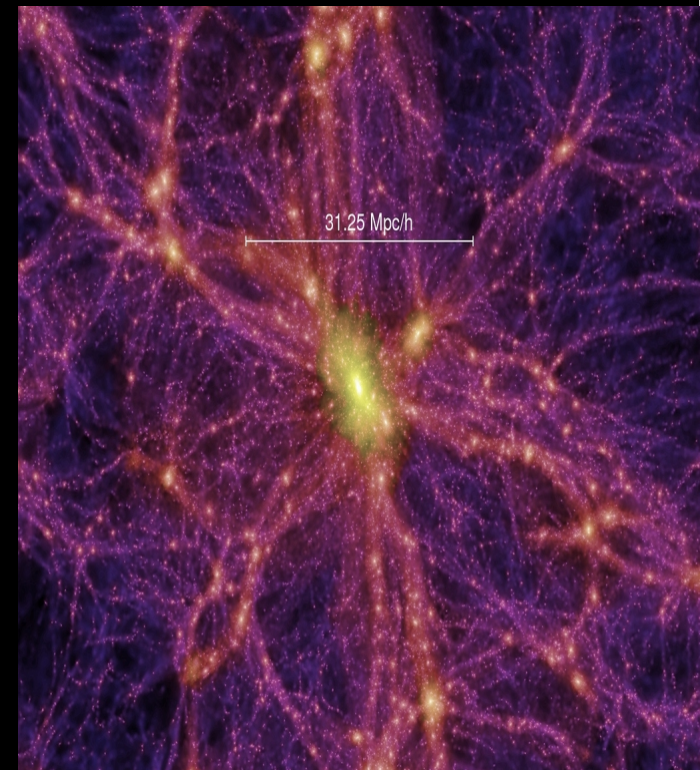
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Shaw et al., 2006

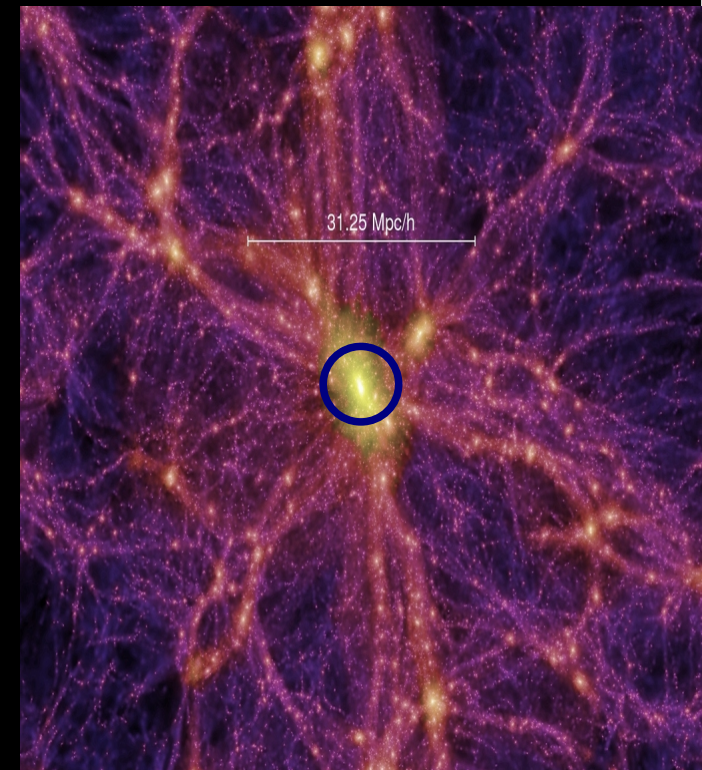
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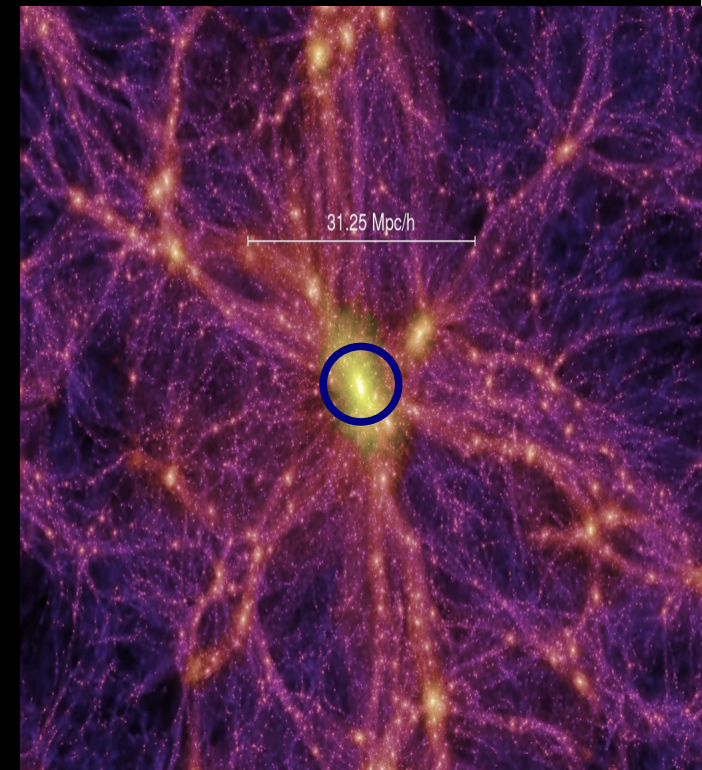
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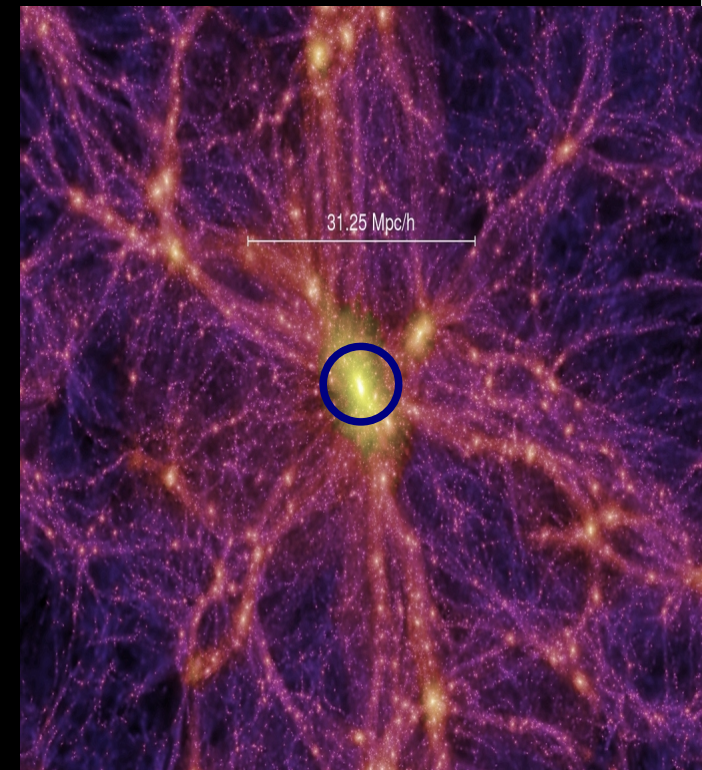
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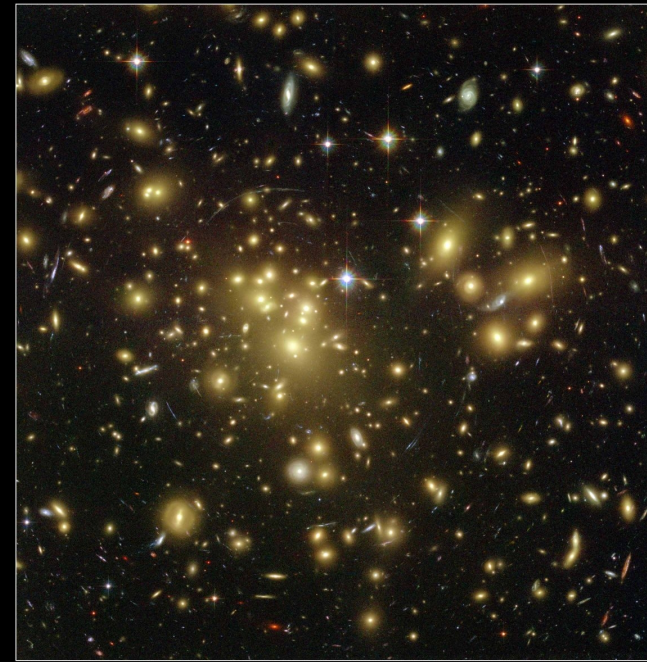
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- ◆ In particular, in lensing we measure a 3D structure with 2D information!
- ◆ Is this a good assumption?



# Why triaxiality?

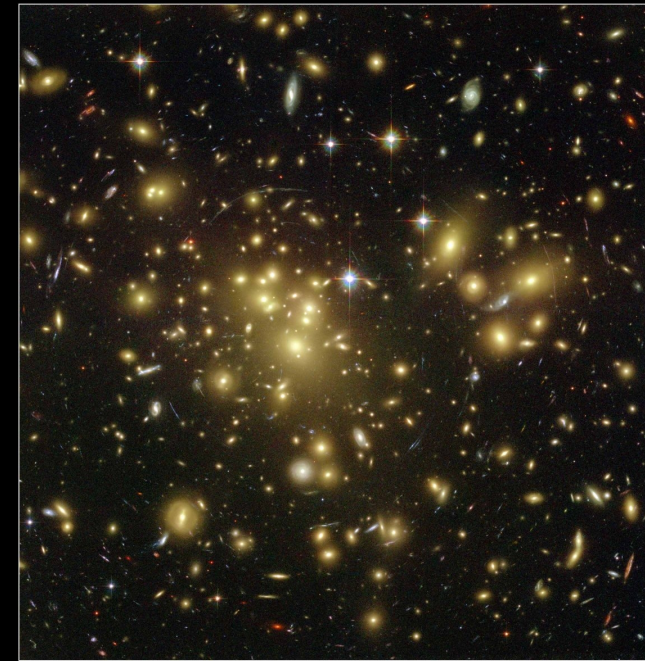
- LCDM simulations predict a  $C-M_{200}$  relation: very massive clusters have low concentrations ( $M = 10^{15} M_{\text{solar}}, C \sim 4$ )



**Galaxy Cluster Abell 1689**  
Hubble Space Telescope • Advanced Camera for Surveys

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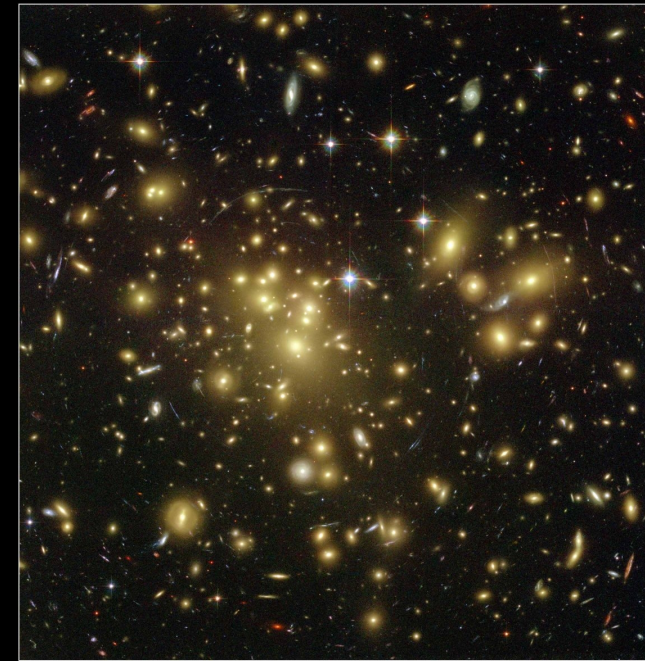
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NASA, N. Benitez (JHU), T. Broadhurst (The Hebrew University), H. Ford (JHU), M. Clampin (STScI), G. Hartig (STScI), G. Illingworth (UCO/Lick Observatory), the ACS Science Team and ESA  
STScI-PRC03-01a

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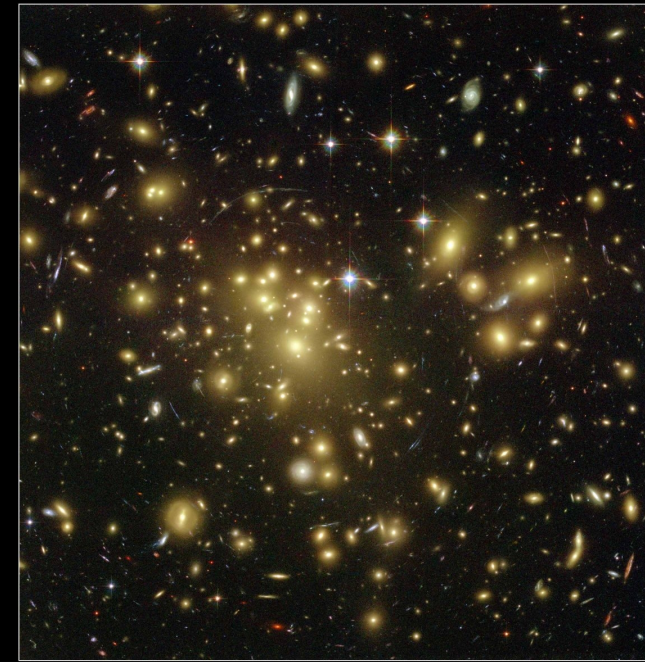
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- ◆ Problems with CDM? Systematics?
- ◆ Some groups have examined the impact of this in specific cases (e.g. Oguri et al. 2005, Gavazzi 2005)



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# Triaxial Dark Matter Halos

$$\rho(R) = \frac{\delta_c \rho_c(z)}{R/R_s (1 + R/R_s)^2}$$

$$R^2 = \frac{X^2}{a^2} + \frac{Y^2}{b^2} + \frac{Z^2}{c^2} \quad (a \leq b \leq c = 1)$$

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$$M_{200} = \frac{800\pi}{3} ab R_{200}^3 \rho_c$$

Triaxial

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$$r_s = R_s (abc)^{1/3}$$

Triaxial

Effective  
spherical



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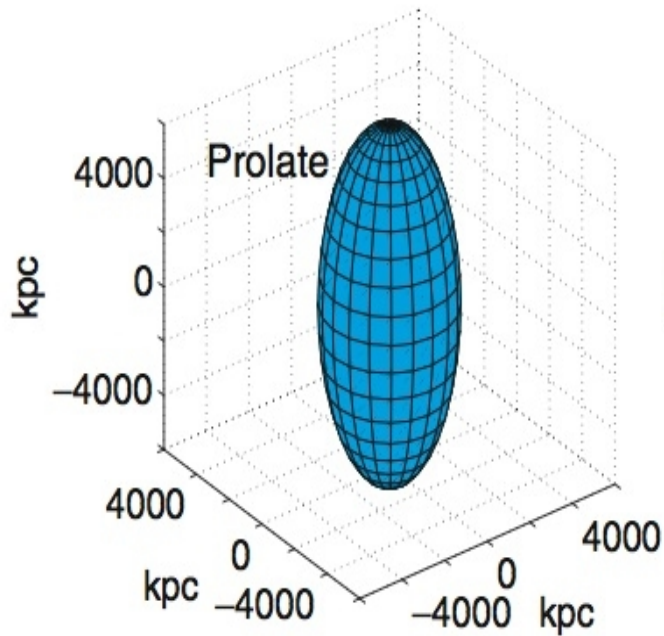
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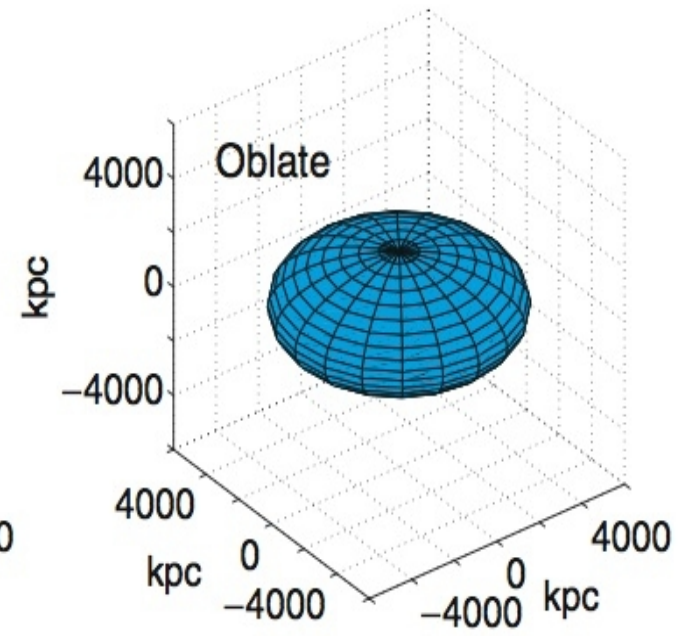
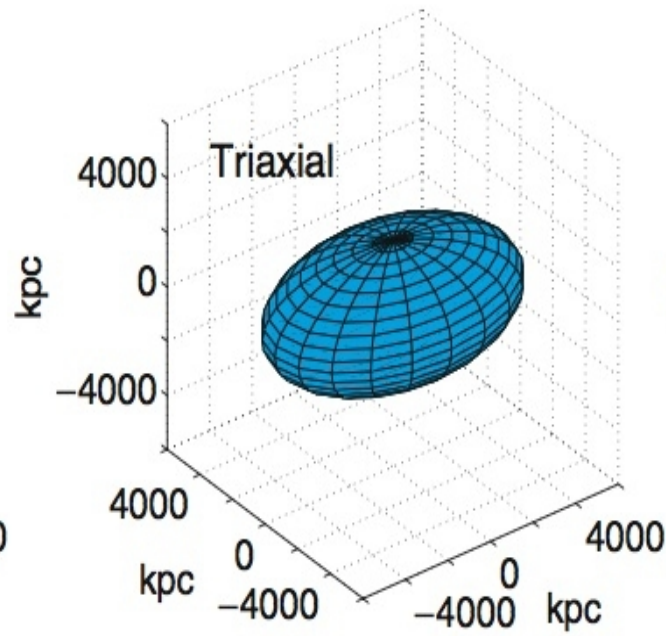
Effective  
spherical

# Triaxial Dark Matter Halos



$$a = b = 0.4$$

“Cigar”



$$a = 0.4, b = 1.0$$

“Pancake”

# Triaxiality: what impact in weak lensing?

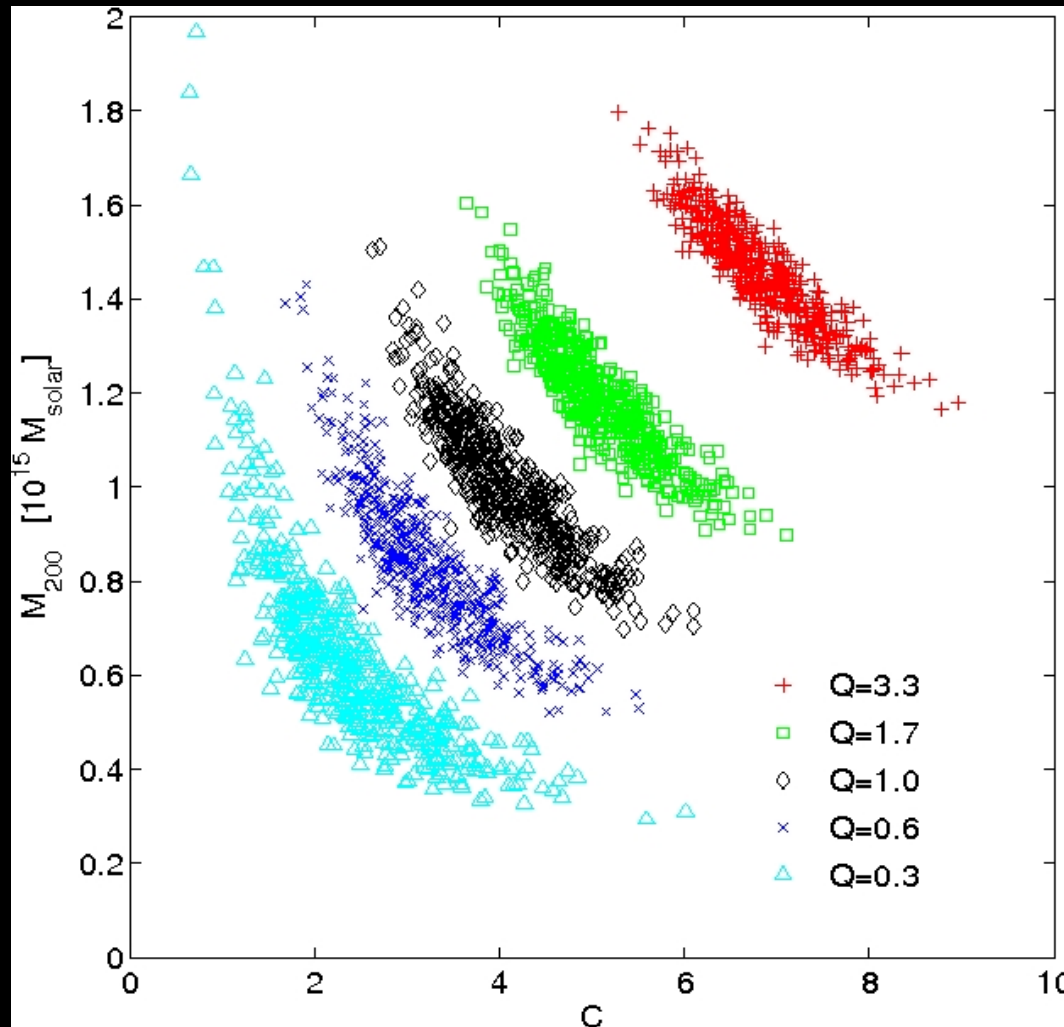
- ◆ Simulate weak lensing through symmetric prolate and oblate halos
  - ◆ (this is the computationally tricky bit – see Keeton 2001; Jing & Suto 2002; Oguri, Lee & Suto 2003; Corless & King 2007)

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- ◆ Simulate weak lensing through symmetric prolate and oblate halos
  - ◆ (this is the computationally tricky bit – see Keeton 2001; Jing & Suto 2002; Oguri, Lee & Suto 2003; Corless & King 2007)
- ◆ Fit spherical NFW models using a standard maximum likelihood technique to the resulting catalogues of lensed objects to obtain estimates of mass and concentration

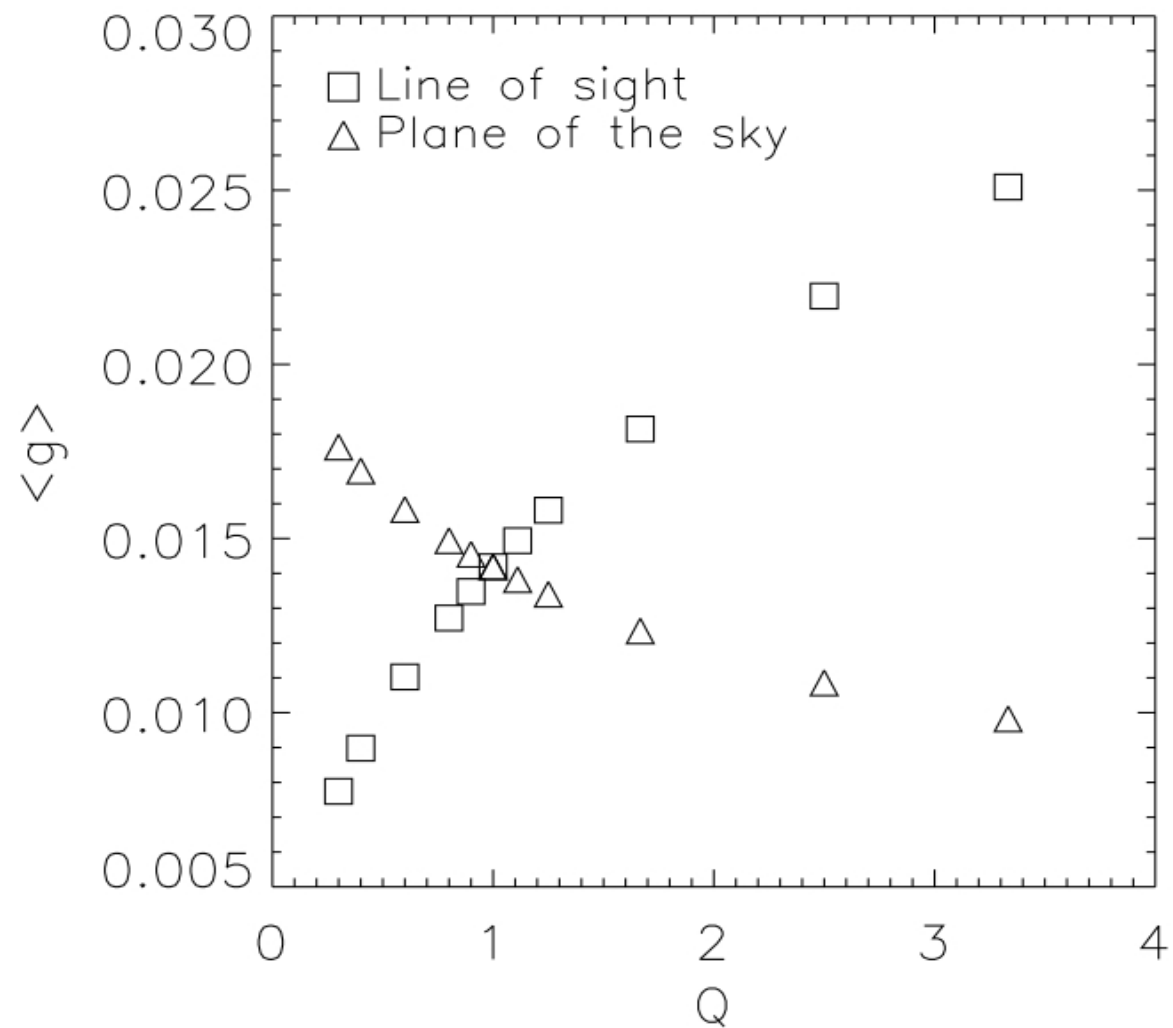
# A Significant Impact!

$M_{200} = 10^{15} M_{\text{solar}}$ ,  $C=4$ , Line of Sight

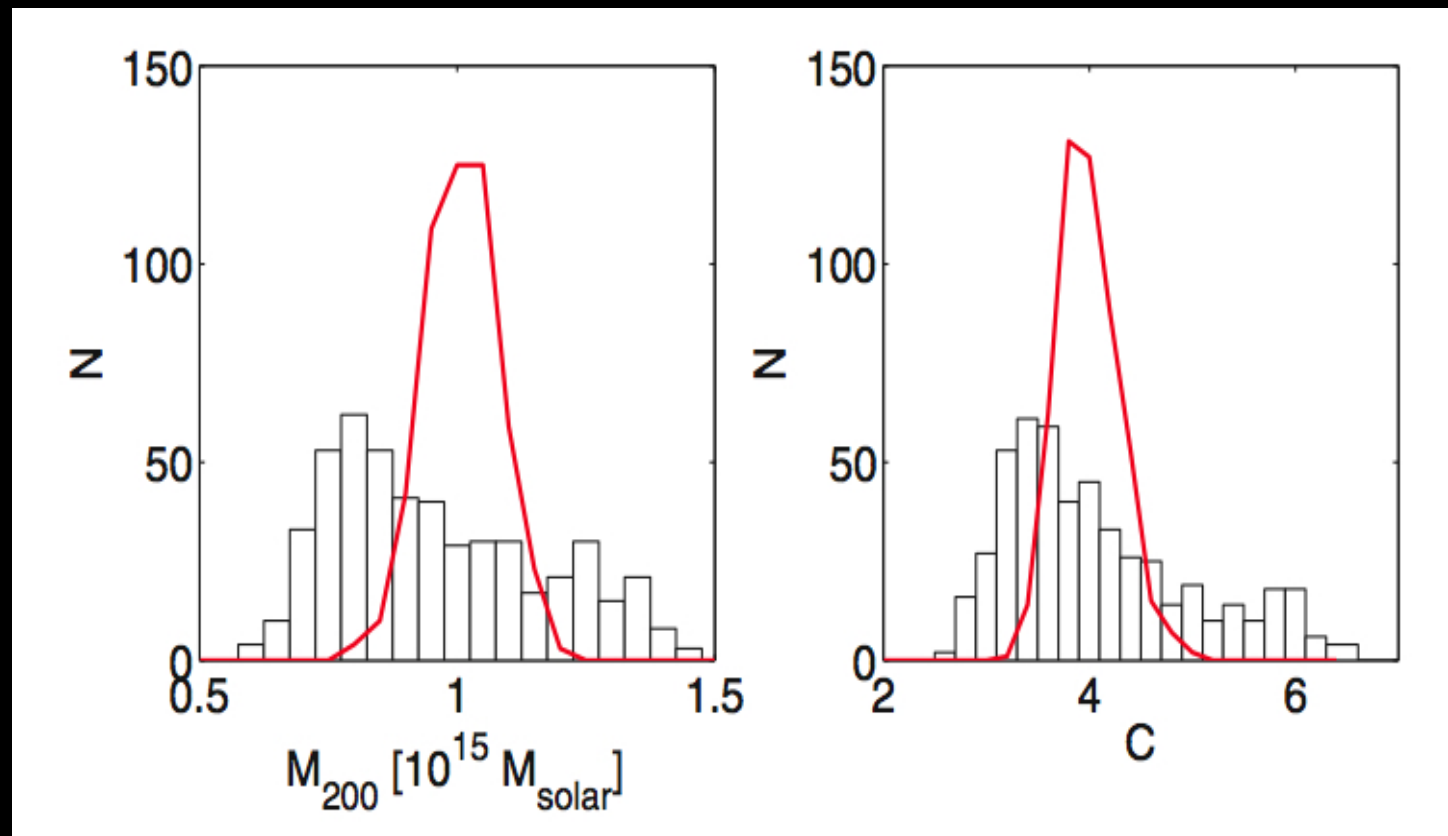


- Errors in mass of up to **~40%**
- Errors in concentration of up to a **factor of 2**

# Lensing efficiency of triaxial halos



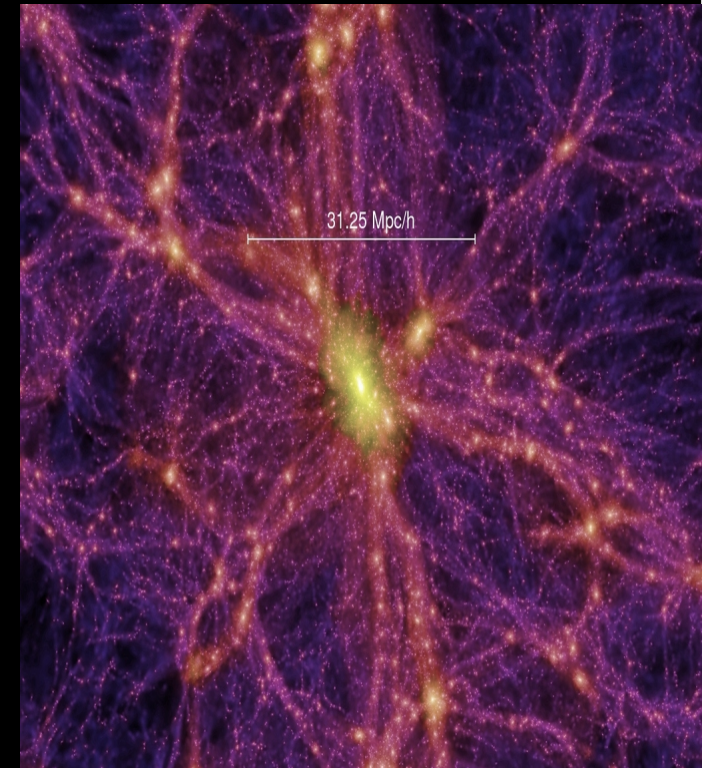
# Populations of triaxial halos



$$M_{200} = 10^{15} M_{\text{solar}}, C=4, \text{Prolate } a = b = 0.4$$

So...

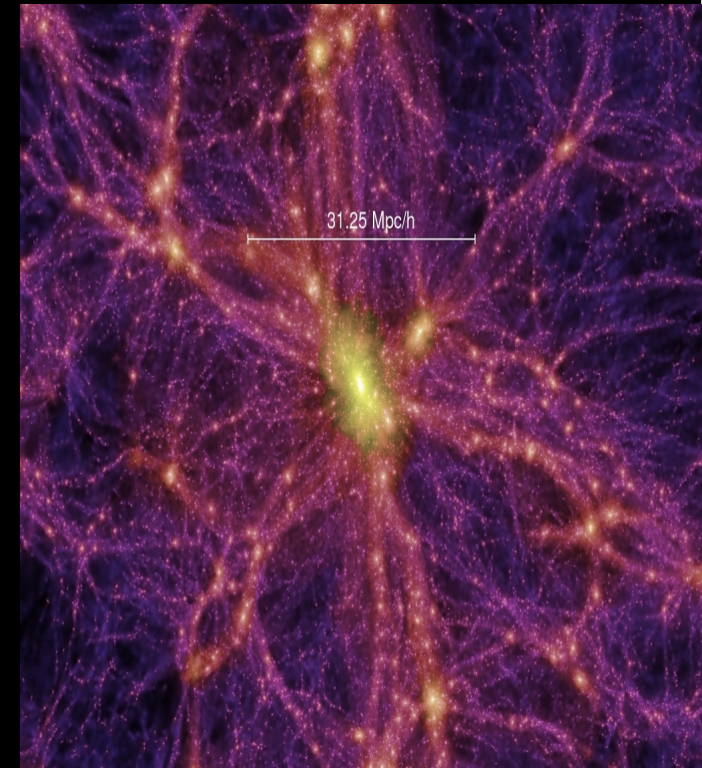
- ◆ Triaxiality is important – can cause significant errors in some cases, small errors in all cases
- ◆ Some very triaxial structures are the most efficient lenses





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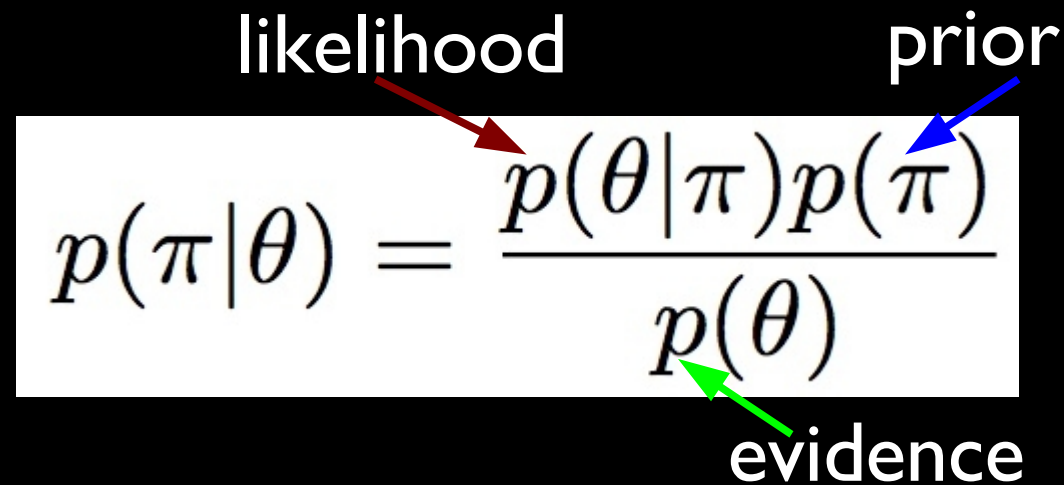
- ◆ Triaxiality is important – can cause significant errors in some cases, small errors in all cases
- ◆ Some very triaxial structures are the most efficient lenses
- ◆ Can we fit triaxial models to lensing data?
- ◆ An intrinsically underconstrained problem: 3 axes to constrain in the model with 2 axes of observed data



# Fitting triaxial mass models

## Markov Chain Monte Carlo

A “guided” random walk that maps complex posterior probability distributions by preferentially sampling regions of high probability, but is free to move downhill to lower probabilities to move between peaks

$$p(\pi|\theta) = \frac{p(\theta|\pi)p(\pi)}{p(\theta)}$$


likelihood

prior

evidence

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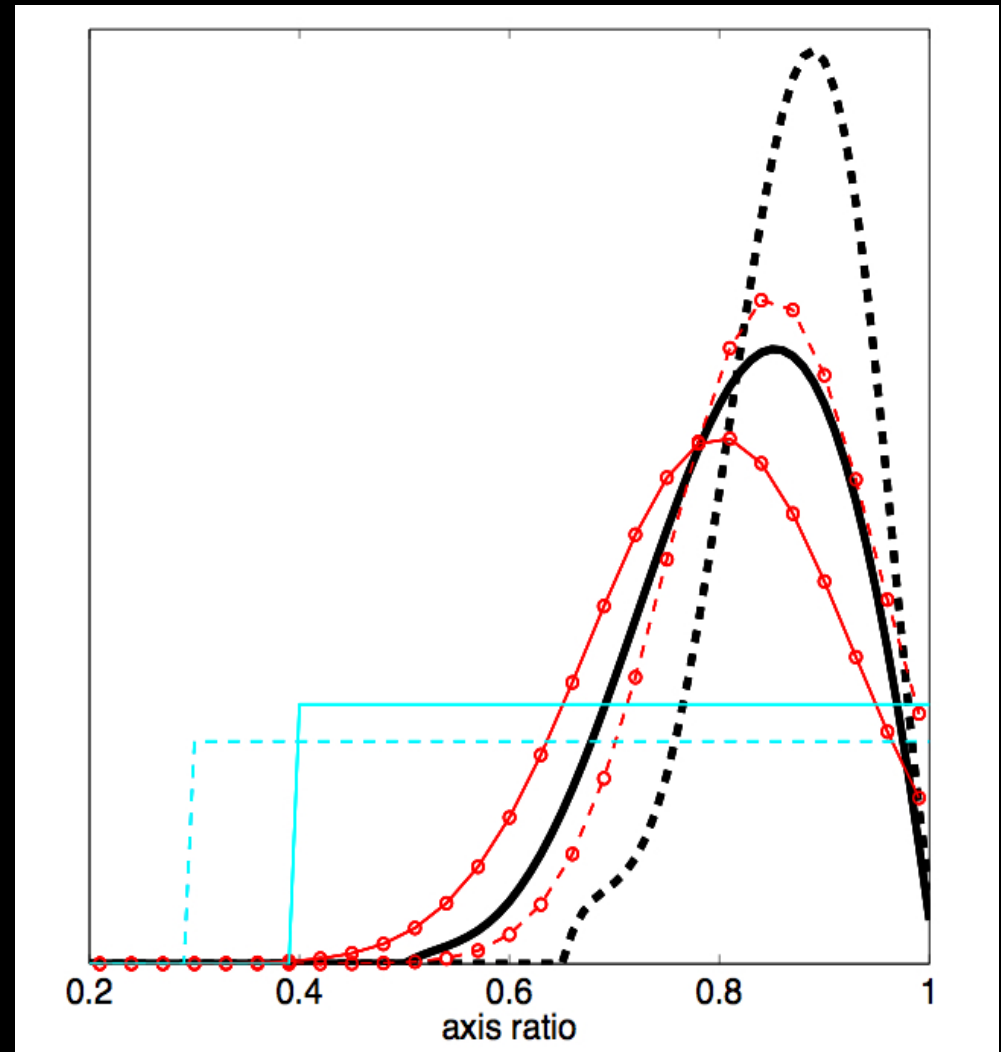
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# Choice of Prior

A fundamentally underconstrained problem!

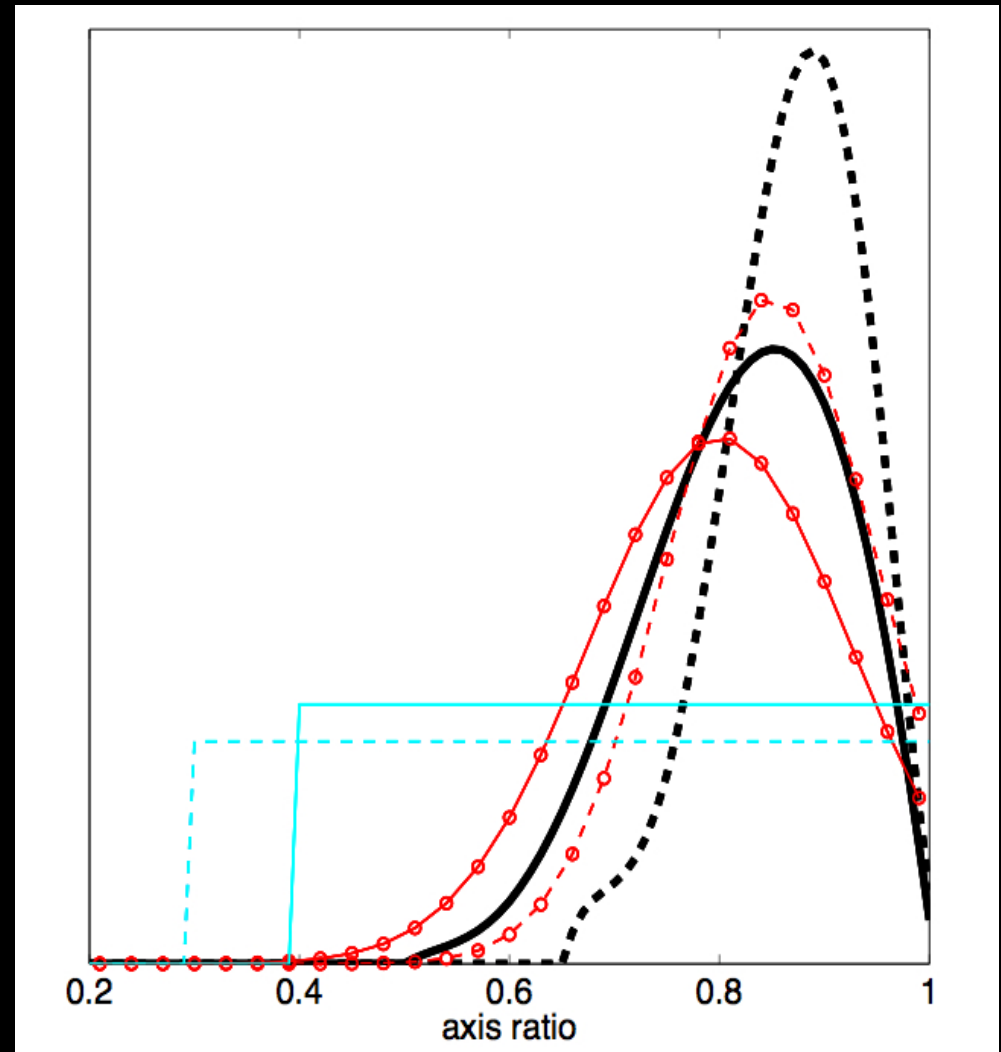
- ◆ Flat
- ◆ Simulations
- ◆ Spherical



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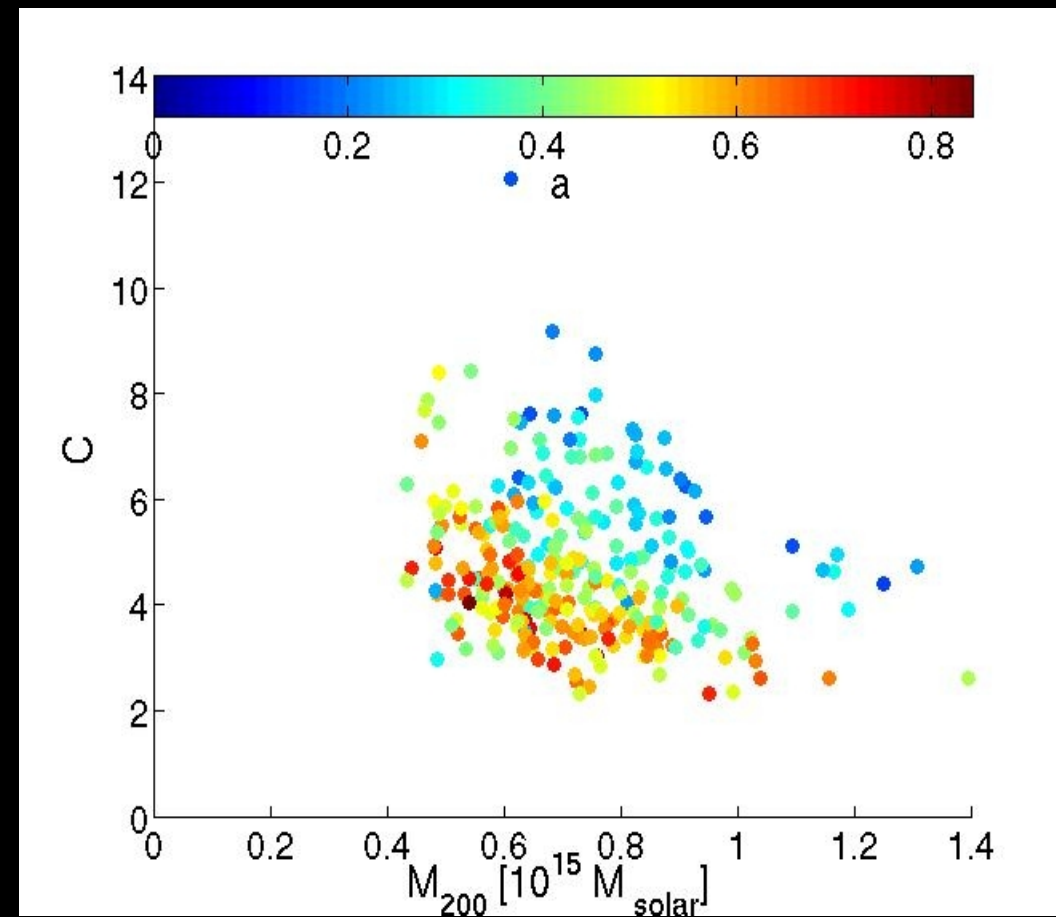
- ◆ Flat
- ◆ Simulations
- ◆ Spherical
- ◆ Mass
- ◆ Lens efficiency



# MCMC

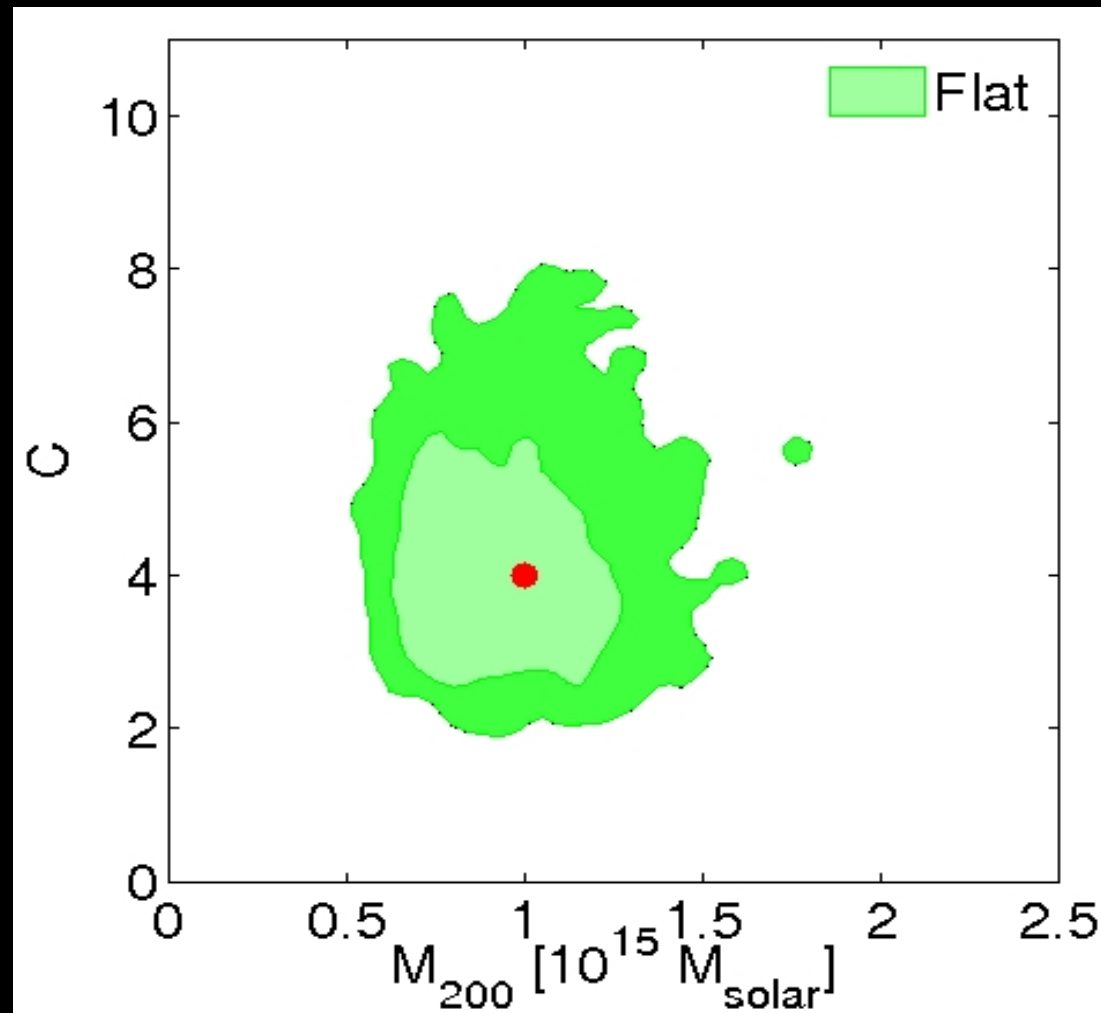
Can define the full posterior probability distribution of triaxial models, giving us true(r) error contours!

Probability  
proportional  
to density of  
points



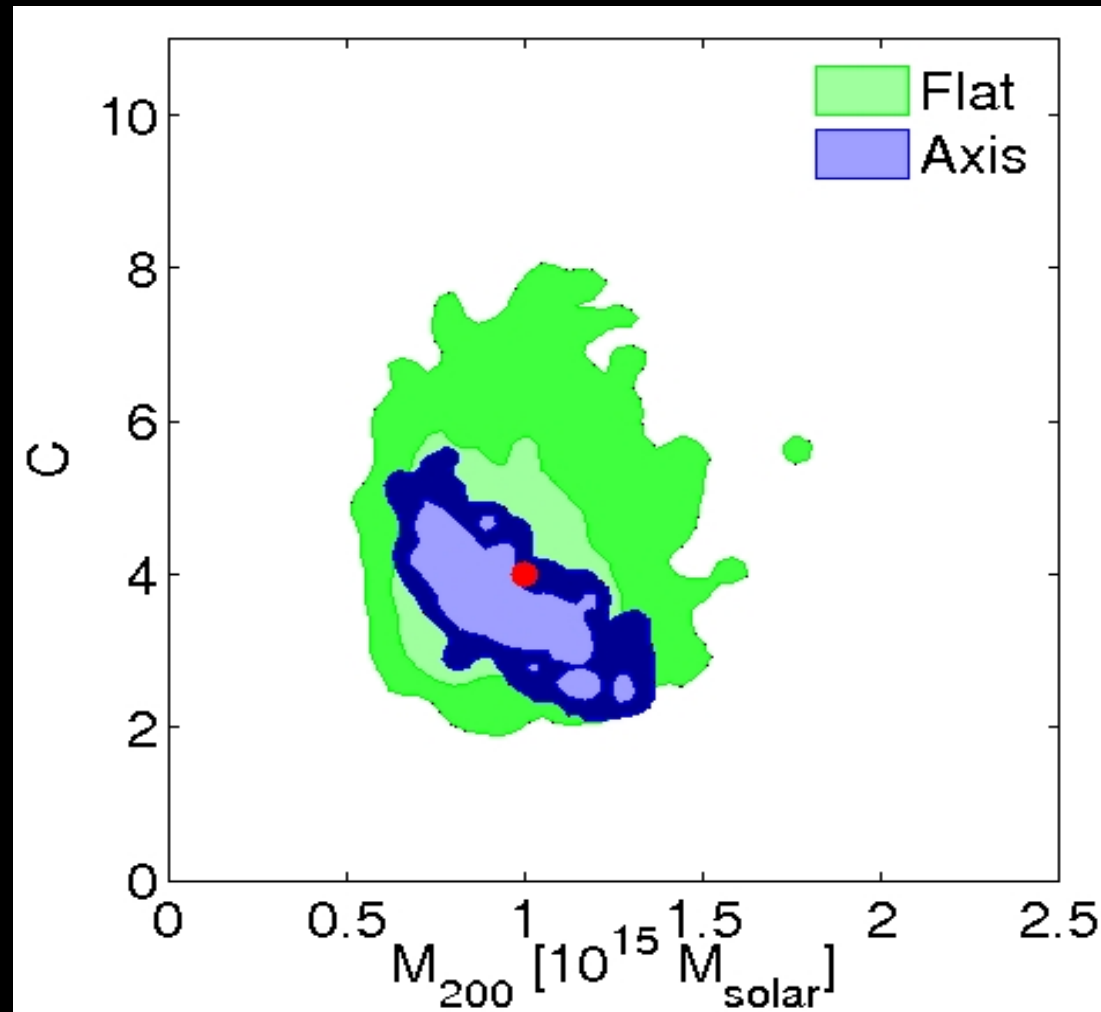
MCMC:  $C=4$ ,  $M=10^{15} M_{\text{solar}}$ ,  $a=.44$ ,  $b=.63$

Flat Prior on Axes



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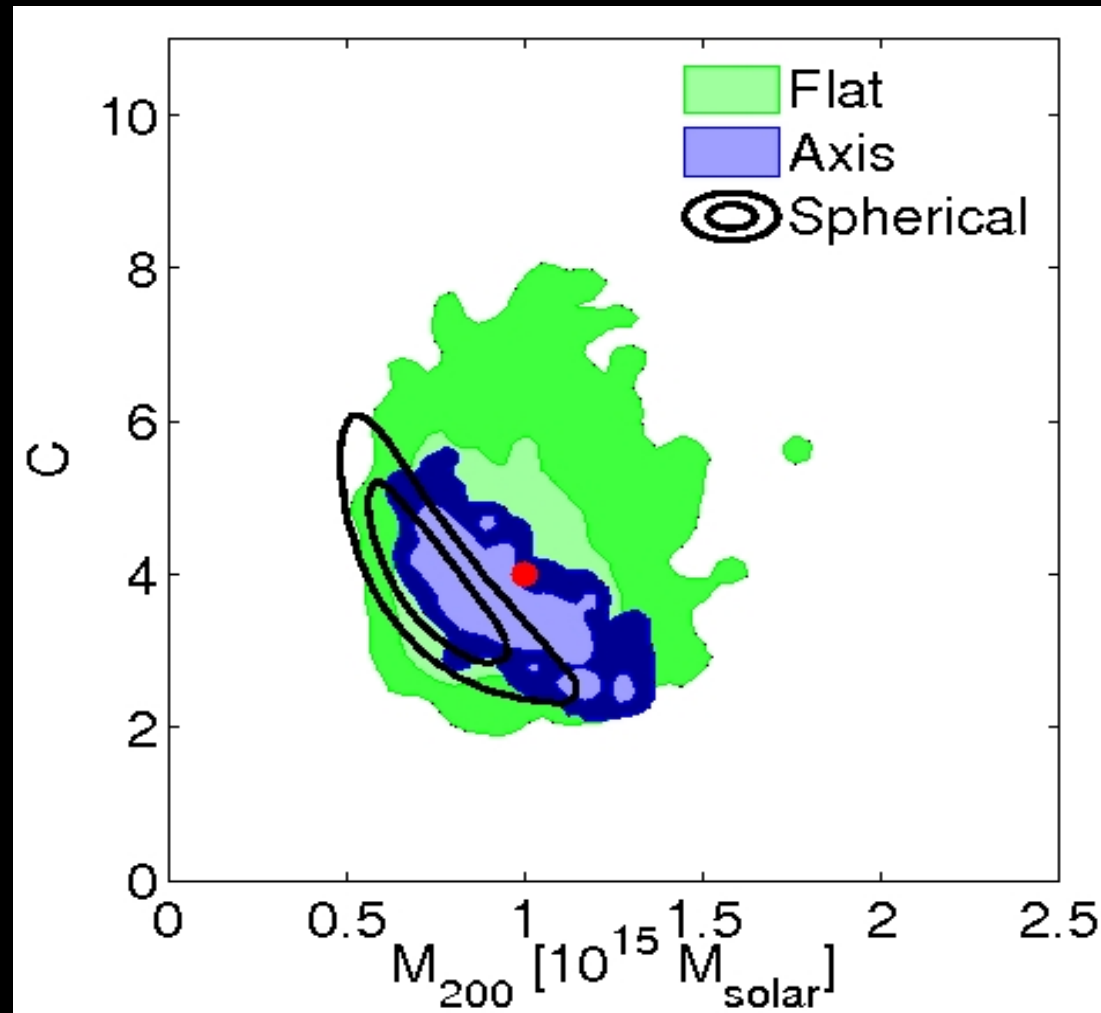
## Shaw Prior on Axes



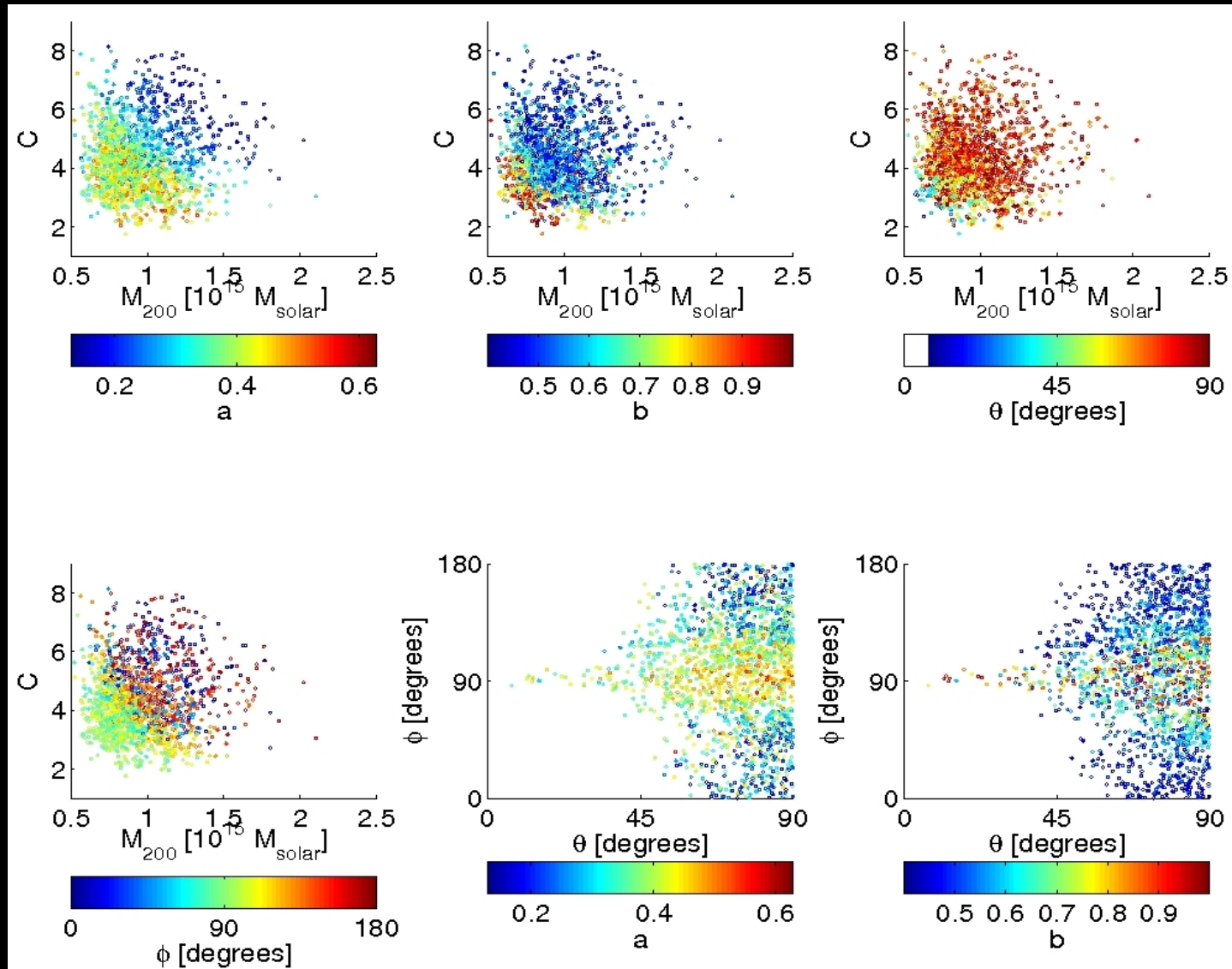


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## Spherical Prior on Axes



**MCMC:  $C=4$ ,  $M=10^{15} M_{\text{solar}}$ ,  $a=.44$ ,  $b=.63$**



# Statistical Performance

## Confidence Contours under various priors

Prior	68%	95%
Flat	86	99
<b>Shaw</b>	<b>66</b>	<b>94</b>
Axis	70	96
Mass	86	99
Spherical	53	81
Effective Spherical Parameterisation		
Flat	84	99
Shaw	61	89
Spherical	56	82

# Statistical Performance

## Mean population values under various priors

Prior	$M_{200}[10^{15} M_{\odot}]$	$C$
Original Population	1.00	4.0
Flat	1.12	4.7
Shaw	1.04	4.2
Axis	1.04	4.3
Mass	1.07	4.7
Spherical	1.02	4.1
Effective Spherical Parameterisation	$m_{200}$	$C_{sph}$
Original Population	0.986	3.98
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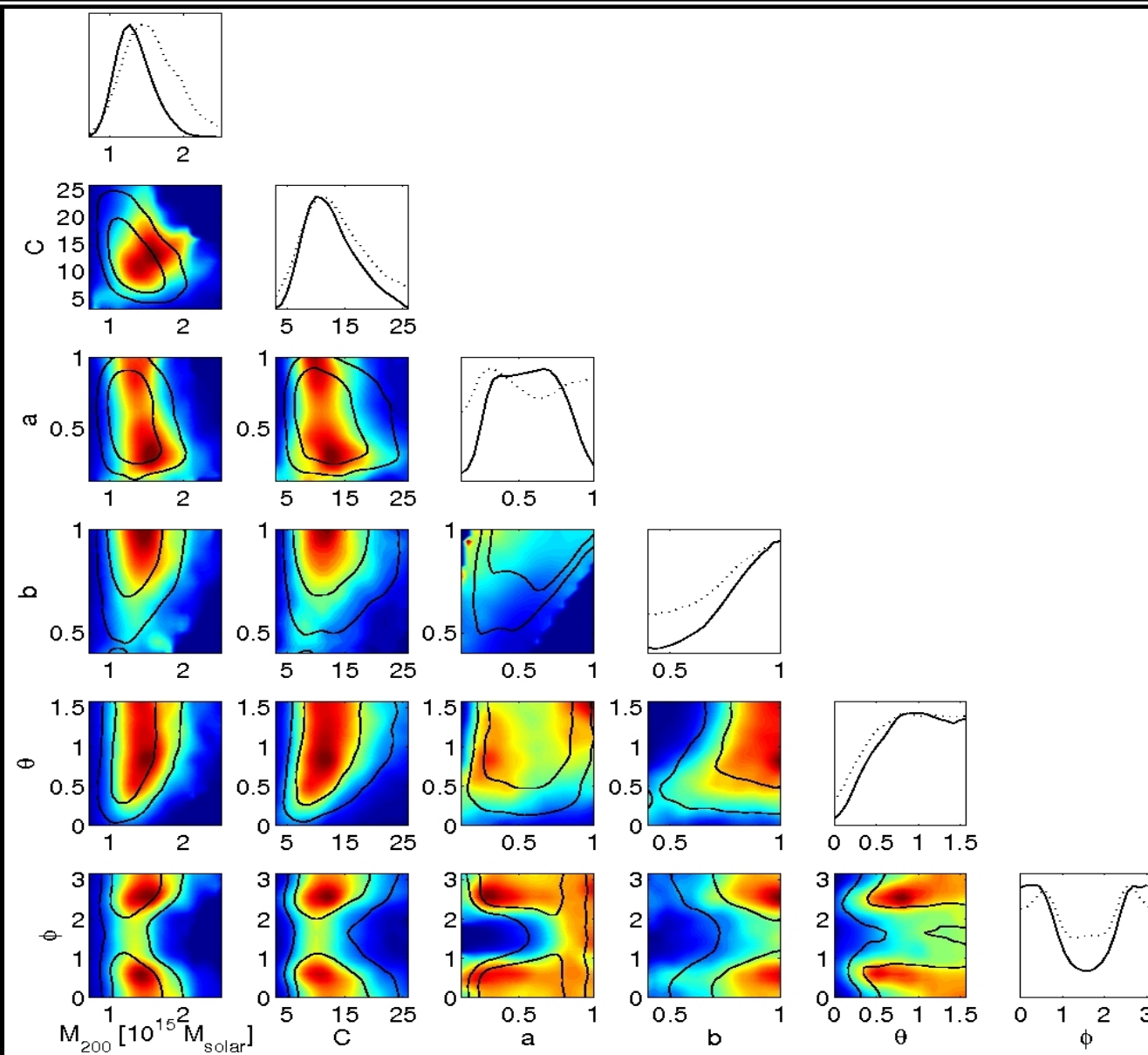
  

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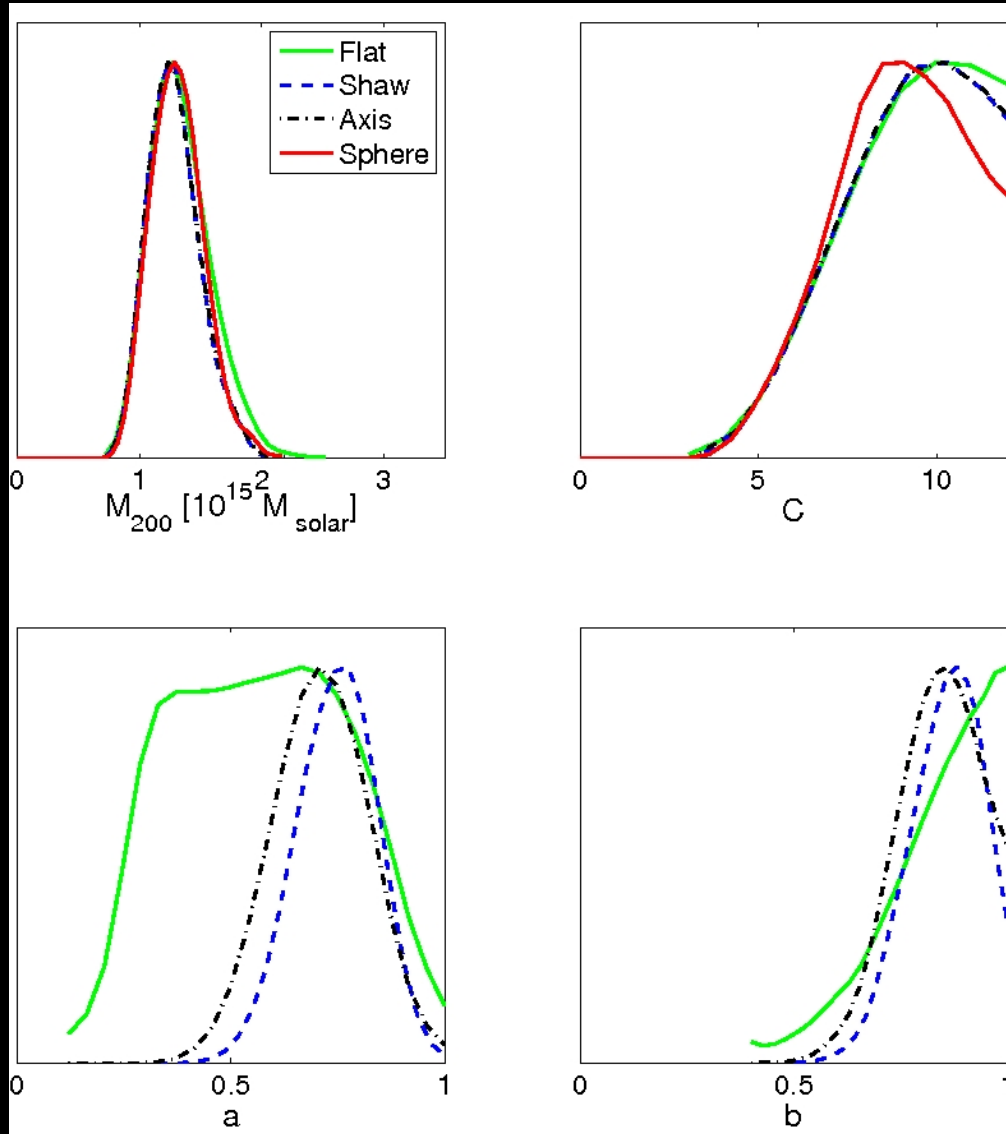
**Triaxial model under sensible prior is best**

But, spherical model may be ok for averages  
(better for masses than concentrations)

# Abell 1689: Flat Axis Prior



# Parameter distributions: Abell 1689



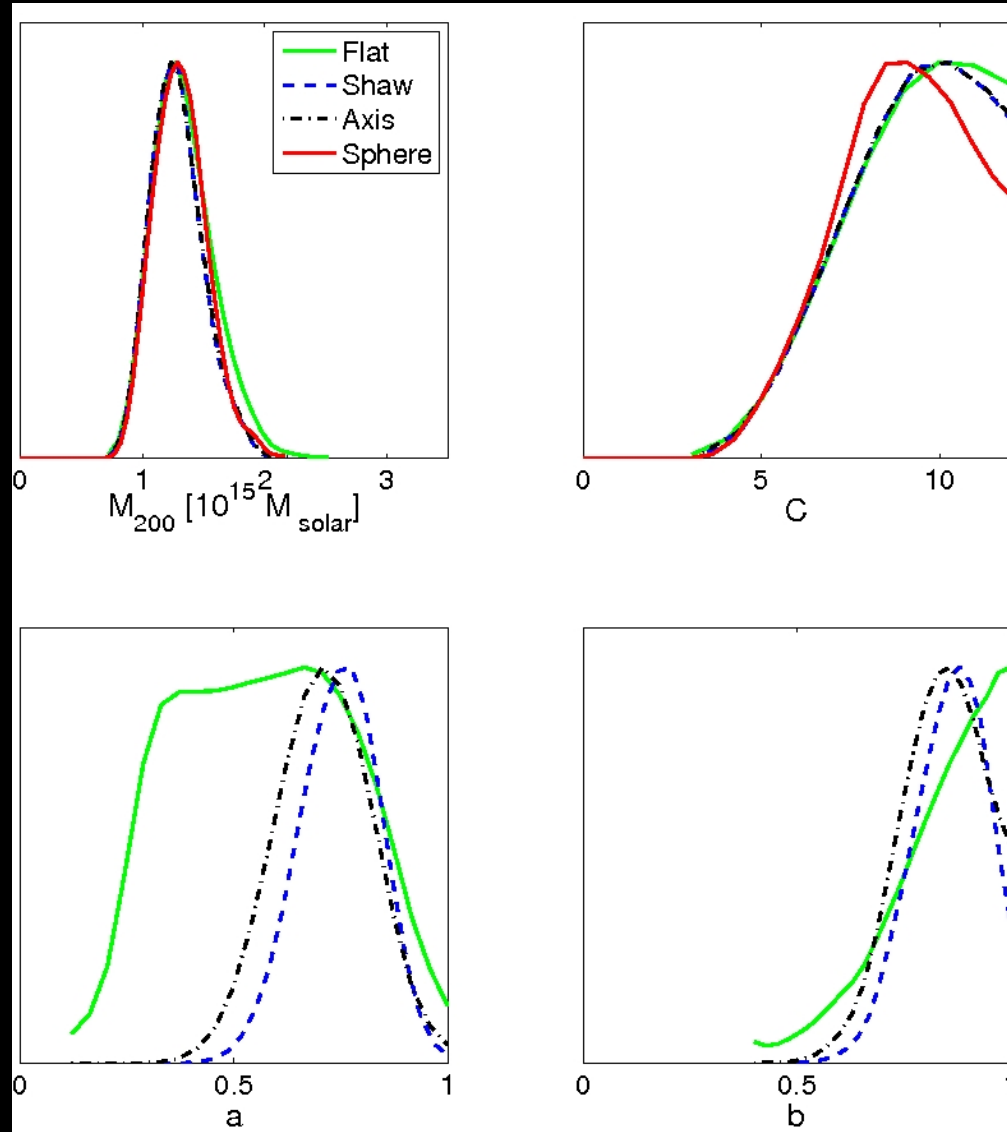
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 M_{200} &= (1.3 \pm 0.2) \times 10^{15} M_{\odot} \\
 C &= 13_{-4}^{+5} \\
 a &= 0.57_{0.12}^{+0.11} \\
 b &= 0.83_{-0.04}^{+0.17}
 \end{aligned}$$

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 C &= 12 \pm 4 \\
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 M_{200} &= (1.32 \pm 0.09) \times 10^{15} M_{\odot} \\
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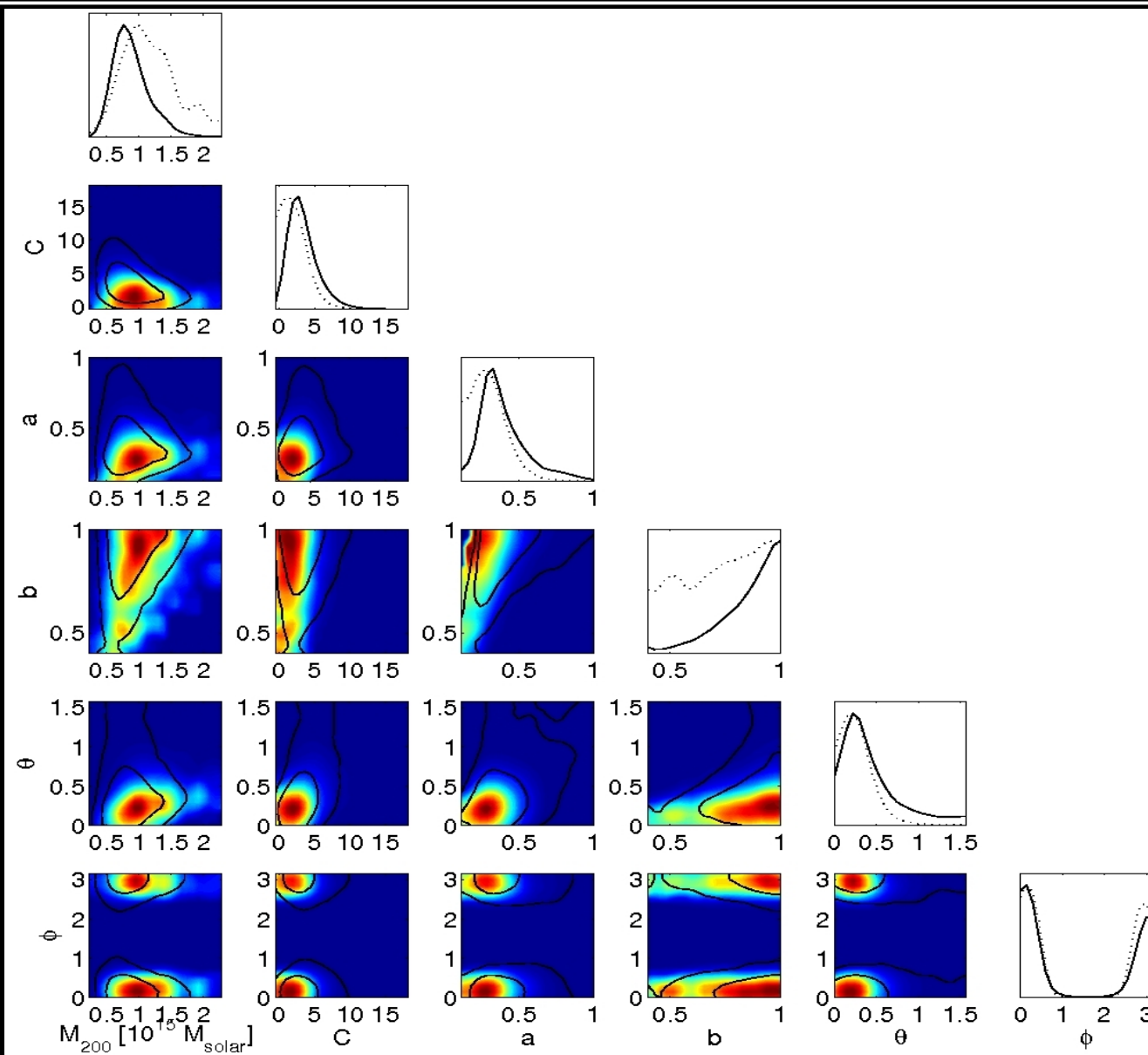
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Better Prior?

# Abell 2204: Flat Prior



# Conclusions

- ◆ MCMC method gives mass estimates and errors that reflect (more) **realistic** triaxial dark matter structures
- ◆ No more quoting spherical lensing results with small errors – though on average, spherical masses may be ok
- ◆ Combined methods (X-ray, SZ, dynamical, SL) are necessary to improve error constraints
- ◆ *Ongoing work*
  - ◆ *Better choice of priors*
  - ◆ *Mass functions*
  - ◆ *Combination methods*